Exploring the understanding of the number *pi*: A case of pre-service secondary school mathematics teachers at a selected institution of higher learning in Namibia

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Introduction

Pi (π) plays a crucial role in circle geometry as well as other areas of Mathematics and Science. Despite the role it plays, its meaning and significance in Mathematics and Science is often underscored.

Most of the high school leavers use this number but they are not aware of what it means and its significance in Mathematics and Science. Many learners in the Namibian high schools are aware of the existence of pi; however they do not really know what it means. This study was therefore deemed necessary in the light of emphasizing deep learning of Mathematics as opposed to the rote learning that is based on the traditional education among first year secondary school pre-service teachers of Mathematics.

In this paper the researchers assessed the understanding of pi among the first year secondary school pre-service teachers of Mathematics on their Mathematical experiences of pi. Furthermore, the researchers also sought to relate the understanding of pi among pre-service first year secondary school pre-service teachers of Mathematics to their academic performance in circle geometry.

Consequently this study sought answers to the following questions:
What meaning do the first year secondary school pre-service Mathematics teachers attribute to pi?
To what extent do the first year secondary school pre-service Mathematics teachers possess understanding of pi?

Literature review

Currently there is no agreed upon and precise meaning of pi. Posamelner and Lekman (2004) define pi from a historical point of view that historically, *pi* is the numerical relationship between the diameter and circumference of a circle. Posamelner and Lekman (2004) further allude to *pi* as a geometric constant thereby operationally defining geometry as the study of drawn figures.

Pi has a rich history. Its history can be traced back to at least four thousand years, often with mathematicians attempting to extend their understanding of π, by calculating its value to a high degree of accuracy (Beckman, 1986). Other literature such as Boyer and Merback (1991) indicated that before the 15th century, mathematicians such as Archimedes and Liu Hui used geometrical techniques, based on polygons, to estimate the value of π. Eymard and Lafon (1999) on the other hand state that starting around the 15th century, new algorithms based on infinite series revolutionized the computation of π, and were used by mathematicians including Madhava of Sangamagrama, Isaac Newton, Leonhard Euler, Carl Friedrich Gauss, and Srinivasa Ramanujan in their calculations.

The common idea among the foregoing literature and also by Blamer (1999); Lay-Tong (1986) opines that the number **π** is a mathematical constant that is the ratio of a circle's circumference to its diameter. The literature further state that *pi* is a constant and is approximately equal to 3.14159. *Pi* is represented by the Greek letter "π" since the mid-18th century (Posamelner and Lekman, 2004). Writers such as Blamer (1999); Eymard and Lafon (1999) as well as Posamelner and Lekman (2004) further suggest that the number *pi* (π) is an irrational number, which means that it cannot be expressed exactly as a ratio of two integers (such as 22/7 or other fractions that are commonly used to approximate π); consequently, its decimal representation never ends and never repeats.

The recent developments in the history of *pi* are that is has been utilised in diverse scientific fields of study. For instance, Posamelner and Lekman (2004) noted that the 20th century, mathematicians and computer scientists discovered new approaches that when combined with increasing computational power, extended the decimal representation of π to over 10 trillion (10^{13}) digits. However, the recent studies of *pi* are more record driven rather than being scientifically motivated.

> While scientific applications generally require no more than 40 digits of π, the primary motivation for these computations is the human desire to break records, but the extensive calculations involved have been used to test supercomputers and high-precision multiplication algorithms (Posamelner and Lekman, 2004, p. 131).

Furthermore several individuals have endeavored to memorize the value of π with increasing precision, leading to records of over 67,000 digits (Posamelner and Lekman, 2004). Because its definition relates to the circle, π is found in many formulae in trigonometry and geometry, especially those concerning circles, ellipses, or spheres. It is also found in formulae from other branches of science, such as cosmology, number theory, statistics, fractals, thermodynamics, mechanics, and electromagnetism (Blamer, 1999). It can therefore be argued that the ubiquitous nature of π makes it one of the most widely known mathematical constants, both inside and outside the scientific community. Also, literature (e.g. Blamer, 1999; Eymard and Lafon, 1999; Posamelner and Lekman, 2004) devoted time and effort to the study of Pi leading to a Pi day which is celebrated on *Pi Day* which is every 14th March, where findings and approximations of π to several are shown.

Furthermore, the later literature shows that *pi* is used to get the volume or a surface area of a disc, the circumference of a circle, areas and volumes of cylinders, spheres and hemispheres, etc. It is also used to measure how fast and how powerful a computer is. Because it is well known it can be used to check computer accuracy and if it has a problem in it software or hardware. *Pi* is also used to get the value of trigonometry functions like sine, cosine, tangent, etc. *Pi* also, plays a crucial role in mechanics where it is used to measure circular velocity of rotating objects for instance a truck wheel, motor shafts, engine parts, gears, etc. (Blamer, 1999). *Pi* continues to play an important role in the study of electronics where it aids in the measurement of AC voltage across a coil and a capacitor. In the natural world *pi* is utilized in measurement of ocean waves, light waves, sound waves, and river bends among others.

The Namibian school Mathematics curriculum assumes that *pi* is common knowledge and therefore learners are expected to learn it constructively from interacting with their environment. Findings of this study indicated that high school graduates show low facility of *pi*. Mathematics teachers often do not deem it necessary to incorporate the teaching of *pi* as a concept that should be taught but they rather make use of it when teaching circle geometry and other scientific concepts that involve *pi*.

This study therefore deemed it necessary to investigate whether *pi* is integrated in the high school curriculum in order for learners to pay attention to its value and role in the scientific arena.

Theoretical framework

This study is informed by the constructivist theory. Constructivism is mainly concerned with cognition, the progression of development of thinking and reasoning as a human action by individuals. Jooste (2011) noted that the constructivist theorists such as Piaget and Vygotsky both advocate and exemplify “transactional, relational and contextualized” approaches for considering human development through interaction with the environment. Constructivism further holds the view that learners construct their own understanding of the knowledge (Jooste, 2011).

This study assumed that during the learners’ secondary school years the learners are exposed to instruction regarding the use of *pi*. Therefore the first year secondary school pre-service Mathematics teachers were subjected to a rich environment that enabled them to gain a better understanding of *pi*.

The researchers therefore, opined that the learners construct their Mathematical concepts through their own experiences and prior understanding. This prior understanding should have been acquired from their previous Mathematical background; hence, there was no need for University lecturers to re-teach the concept of *pi* to them. This study therefore sought to gauge the first year pre-service mathematics teachers' understanding of the number *pi*.

Methodology

The study sought to assess the first year secondary school pre-service Mathematics teachers' comprehension of *pi* as a number and its influence in their understanding of circle geometry. To do this the study adopted a sequential mixed methods (qualitative and quantitative) research design. Cresswell (2003) indicates that the use of the dual research i.e. qualitative and quantitative approaches has become a common practice in research and yield comprehensive results in the sense that results obtained through the quantitative method are described thoroughly by the qualitative data obtained from the research participants (Loraine, 1998).

Twenty first year secondary school pre-service Mathematics teachers at Hifikepunye Pohamba Campus of the University of Namibia were randomly elected in order to assess their understanding of *pi*. The random sampling procedure was used in this study because of the large number of first year secondary school pre-service Mathematics teachers because the group was too large to be studied.

Presentation and discussion of the results

Students were asked to indicate their grades in Mathematics at Grade 12. Table 1 shows their responses.

Table 1: Grades of the first year secondary school Mathematics pre-service teachers in the Mathematics Grade 12 examinations (N=20)

Grade	No. of students
A	5
B	9
C	3
D	1
E	2
Total	**20**

From Table 1, 17 of the first year secondary school pre-service Mathematics teachers had quite a good mathematical background. Therefore, one expects them to possess a better understanding of *pi*. Students were asked to indicate at what grade they first encountered *pi*, Table 2 shows their responses.

Table 2: The grade level when the first year secondary school pre-service Mathematics teachers encountered *pi* (N = 20)

Grade level	No. of students
Grade 7	6
Grade 8	8
Grade 9	2
Grade 10	4
Total	20

Table 2 indicates that the concept of *pi* is developed as early as Grade 6 in Namibian schools. This finding seems to indicate that the participants in this study had interacted and used the concept *pi* for over 5 years.

First year secondary school pre-service Mathematics teachers were further asked to indicate the sources of their information about pi. Five (5) participants indicated their Mathematics teacher; 10 indicated from the Mathematics textbooks2 indicated from dictionary, the remaining 3 participants indicated that they did not remember how they acquired the concept of *pi.* Furthermore, the student teachers were asked to explain what they understood by the concept *pi*. Two indicated that *pi* meant $\frac{22}{7}$, 1 said *pi* meant 3.141592654, 3 said that *pi* meant nothing to them, 2 students indicated that pi meant 3.142, 3 students said *pi* meant 3.14. Five student Mathematics teachers indicated that *pi* was a ratio of the circle's circumference to its diameter. Another three student Mathematics pre-service teachers indicated that they had no idea what *pi* meant. From the results only 5 Mathematics preservice teachers had an accurate meaning of *pi*. This result points to either poor instruction at secondary school level or that the students had not grasped the meaning of *pi* prior to their admission to university to be trained as Mathematics teachers. Students were also asked to indicate the relationship between ***pi*** and $\frac{22}{7}$; their responses are given in Fig 1.

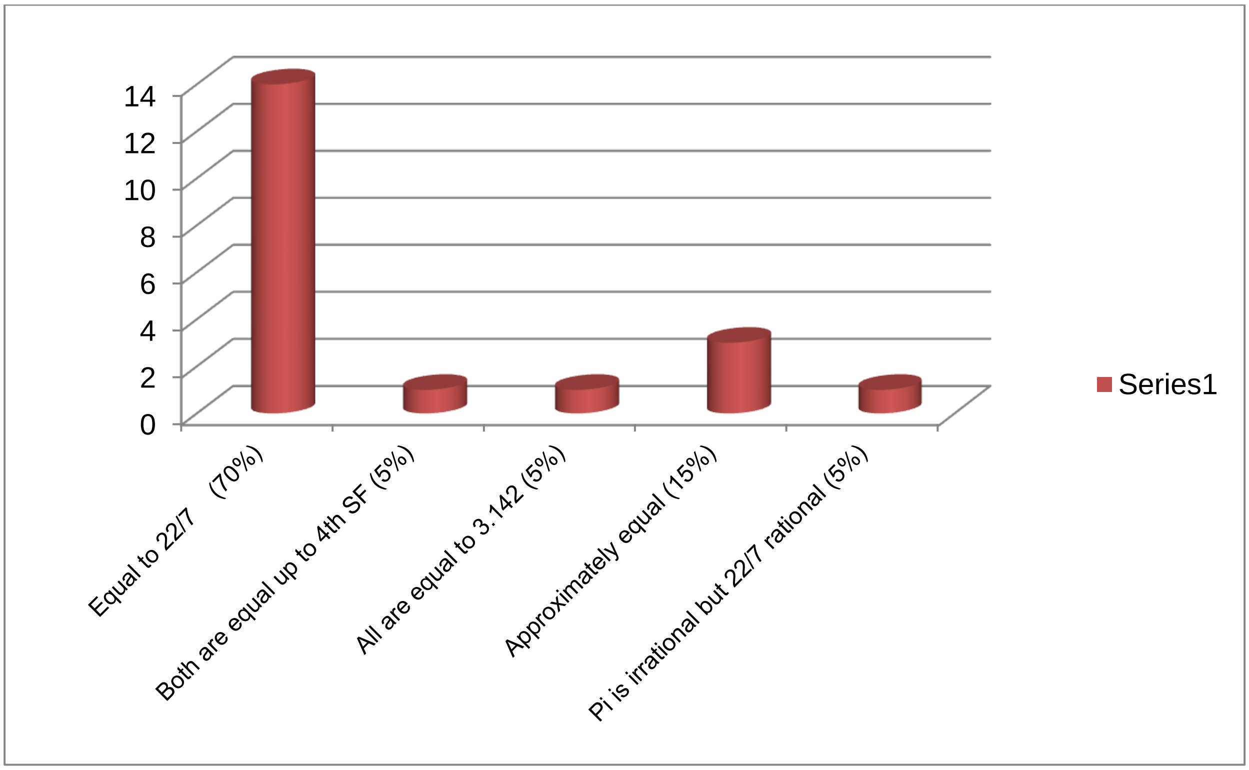

Figure1: Students' responses on the relationship between *pi* and $\frac{22}{7}$

Fig 1 indicates that 14 of the Mathematics pre-service teachers of indicated that there was no difference between *pi* and $\frac{22}{7}$. This is a common misconception in Mathematics, and supports Posamelner and Lekman's (2004) findings that research has proven that approximately 75% of high school leavers possess misconceptions about *pi* and $\frac{22}{7}$ Moreover 3 of the pre-service teachers indicated

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that *pi* was approximately equal to $\frac{22}{7}$, while one indicated that *pi* and $\frac{22}{7}$ were equal up to 4 significant figures. The pre-service Mathematics teachers were also asked to indicate what type of a number *pi* is. The pre-service Mathematics teachers responded according to the supplied options which included: *improper fraction, proper fraction, whole number, decimal; and mixed number*. Their responses are presented in Figure 2.

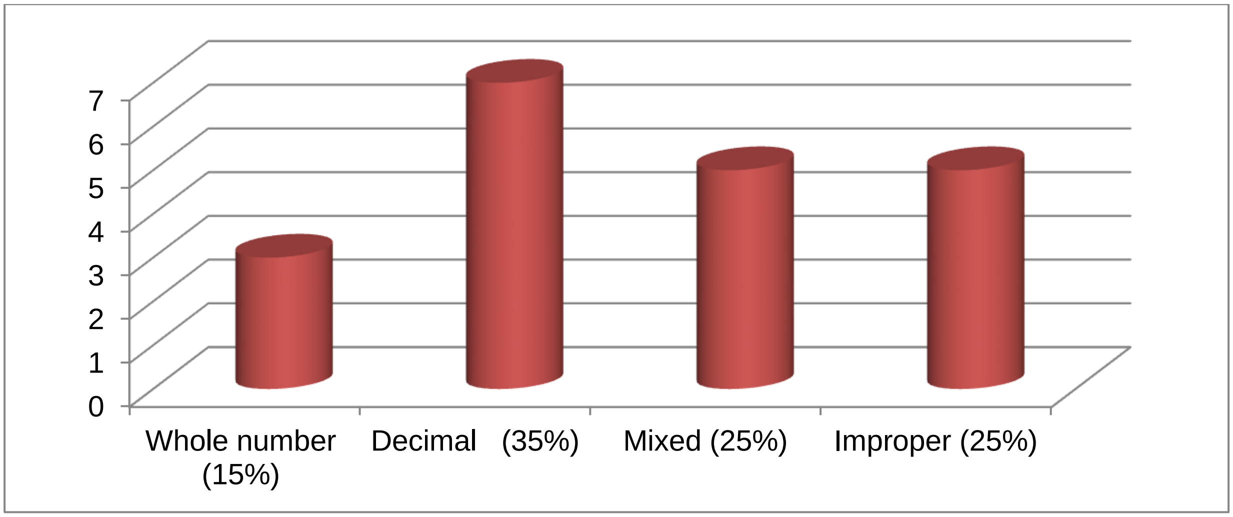

Figure 2: Pre-service Mathematics teachers' responses regarding what type of number *pi* is

Figure 2 shows that the pre-service Mathematics teachers lacked basic understanding of number theory. Thirteen of the participants did not know that *pi* is a none terminating or repeating decimal. It is interesting to note that these participants were expected to join the teaching profession in four years' time and if their misconceptions regarding the concept pi are not. The other issue that was addressed by this study was whether *pi* is a rational or an irrational number. Figure 3 shows the responses of student teachers regarding the question of rationality of *pi*.

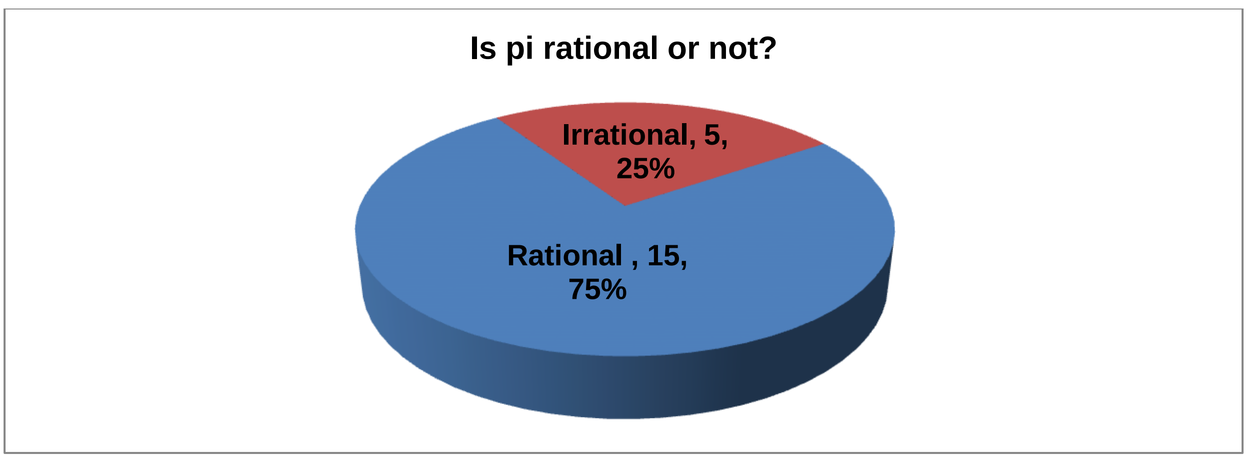

Figure 3: Students' responses on the rationality of *pi*

Fifteen of the pre-service teachers of Mathematics did not possess a better understanding of rational and irrational numbers and therefore could not respond accurately to the rationality of *pi*. It is important that the pre-service Mathematics teachers be remedied on the concepts "rational" and "irrational". Moreover, there is need to include the teaching of number theory to learners in Grades 11 & 12 at secondary school phase so as to remedy the misconceptions they possess on the rationality of numbers. The other issue that was addressed in this study was the perceptions of pre-service teachers on what the exact value of *pi* is. To date mathematicians and scientists have not managed to give conclusively the value of *pi* to a finite number of decimal places (Posamelner and Lekman, 2004). The Mathematics student teachers were asked to give the value of *pi*. Their responses are shown in Figure 4.

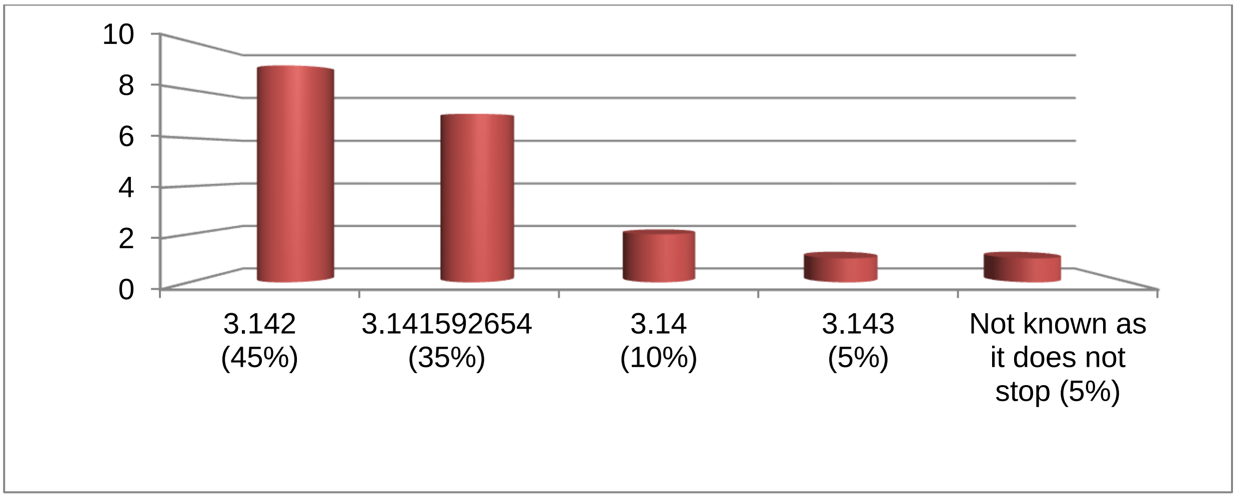

Fig. 4. Pre-service Mathematics teachers' perceptions of the exact value of *pi*

Nineteen pre-service Mathematics teachers did not possess an accurate understanding of the value of *pi (Figure 4)*. This could be attributed to the fact that they did not know what is meant when a number is irrational. Most of the pre-service Mathematics teachers in this study who indicated that *pi* is rational also had failed to give the "exact" value of *pi*.

Conclusion and recommendations

The following conclusions are made on the basis of the data presented in this study.

Most (17) of the pre-service Mathematics teachers possessed good grades in Mathematics (A to C) in their Grade 12 school leaving examinations giving the impression that they had grasped the content given at this level prior to their admission to UNAM. Fourteen of the pre-service Mathematics teachers of had learned and met the number *pi* during their Junior Secondary phase (i.e. Grades 8-10) while 6 had met the concept of *pi* as early as Grade 7. There is need to start teaching Mathematics content at the same grade levels in Namibian schools, so that all learners enter secondary school and tertiary institutions with almost the same content. It is hoped that the implementation of compulsory may address this problem.

Fifteen of the pre-service Mathematics teachers of held the idea that *pi* was an irrational number but could not explain why. Moreover, the pre-service Mathematics teachers did not seem to understand the characteristic features of irrational numbers. This needs to be emphasised in the teaching of school Mathematics.

Nearly all of the pre-service Mathematics teachers (19) in this study lacked an understanding that scientists and mathematicians have not really to date found the exact value of *pi*. The pre-service Mathematics teachers therefore used the approximated values of *pi* and took them to be the exact value. This could be due to the fact that their primary and/or secondary school Mathematics teachers did not probably adequately explain that all values of *pi* used in their mathematical content did not really represent the exact value but were mere approximations.

Most of the students adequately summarised the use of *pi* in Mathematics as to find the area and circumference of circles. However most of the students could not really account for the use of *pi* beyond the mathematical arena. Teachers need to extend the use of numbers beyond the Mathematics classroom. The overall conclusion is that pre-service Mathematics teachers in this study did not possess a better grasp of the concept of *pi*, therefore, Mathematics teacher educators should ensure that pre-service teachers are proficient in basis Mathematics content for their learners to benefit from classroom instruction.

In summary the findings of this study were that first year secondary school pre-service Mathematics teachers did not possess strong understanding of the concept of "pi". About 90% of first year secondary school pre-service Mathematics teachers who demonstrated low level understanding of pi also scored low marks on the circle geometry test. Analysis of the high school curriculum indicated that the concept of pi was not specifically taught in high school Mathematics curriculum and this deficiency could have compromised the first year pre-service mathematics teachers' understanding of *pi*.

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